

NAG Fortran Library Routine Document

G13CDF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

G13CDF calculates the smoothed sample cross spectrum of a bivariate time series using spectral smoothing by the trapezium frequency (Daniell) window.

2 Specification

```
SUBROUTINE G13CDF(NXY, MTXY, PXY, MW, IS, PW, L, KC, XG, YG, NG, IFAIL)
INTEGER          NXY, MTXY, MW, IS, L, KC, NG, IFAIL
real           PXY, PW, XG(KC), YG(KC)
```

3 Description

The supplied time series may be mean and trend corrected and tapered as in the description of G13CBF before calculation of the unsmoothed sample cross-spectrum

$$f_{xy}^*(\omega) = \frac{1}{2\pi n} \left\{ \sum_{t=1}^n y_t \exp(i\omega t) \right\} \times \left\{ \sum_{t=1}^n x_t \exp(-i\omega t) \right\}$$

for frequency values $\omega_j = \frac{2\pi j}{K}$, $0 \leq \omega_j \leq \pi$.

A correction is made for bias due to any tapering.

As in the description of G13CBF for univariate frequency window smoothing, the smoothed spectrum is returned as a subset of these frequencies,

$$\nu_l = \frac{2\pi l}{L}, \quad l = 0, 1, \dots, [L/2]$$

where [] denotes the integer part.

Its real part or co-spectrum $cf(\nu_l)$, and imaginary part or quadrature spectrum $qf(\nu_l)$ are defined by

$$f_{xy}(\nu_l) = cf(\nu_l) + iqf(\nu_l) = \sum_{|\omega_k| < \frac{\pi}{M}} \tilde{w}_k f_{xy}^*(\nu_l + \omega_k)$$

where the weights \tilde{w}_k are similar to the weights w_k defined for G13CBF, but allow for an implicit alignment shift S between the series:

$$\tilde{w}_k = w_k \exp(-2\pi i S k / L).$$

It is recommended that S is chosen as the lag k at which the cross covariances $c_{xy}(k)$ peak, so as to minimize bias.

If no smoothing is required, the integer M , which determines the frequency window width $\frac{2\pi}{M}$, should be set to n .

The bandwidth of the estimates will normally have been calculated in a previous call of G13CBF for estimating the univariate spectra of y_t and x_t .

4 References

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day
 Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

- 1: NXY – INTEGER *Input*
On entry: the length, n , of the time series x and y .
Constraint: $NXY \geq 1$.
- 2: MTXY – INTEGER *Input*
On entry: whether the data is to be initially mean or trend corrected.
 MTXY = 0
 For no correction.
 MTXY = 1
 For mean correction.
 MTXY = 2
 For trend correction.
Constraint: $0 \leq MTXY \leq 2$.
- 3: PXY – *real* *Input*
On entry: the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper.
 A value of 0.0 implies no tapering.
Constraint: $0.0 \leq PXY \leq 1.0$.
- 4: MW – INTEGER *Input*
On entry: the frequency width, M , of the smoothing window as $\frac{2\pi}{M}$.
 A value of n implies that no smoothing is to be carried out.
Constraint: $1 \leq MW \leq NXY$.
- 5: IS – INTEGER *Input*
On entry: the alignment shift, S , between the x and y series. If x leads y , the shift is positive.
Constraint: $-L < IS < L$.
- 6: PW – *real* *Input*
On entry: the shape parameter, p , of the trapezium frequency window.
 A value of 0.0 gives a triangular window, and a value of 1.0 a rectangular window.
 If $MW = NXY$ (i.e., no smoothing is carried out) then PW is not used.
Constraint: $0.0 \leq PW \leq 1.0$ if $MW \neq NXY$.

7: L – INTEGER *Input*

On entry: the frequency division, L , of smoothed cross spectral estimates as $\frac{2\pi}{L}$.

Constraints:

$$L \geq 1,$$

L must be a factor of KC (see below).

8: KC – INTEGER *Input*

On entry: the order of the fast Fourier transform (FFT) used to calculate the spectral estimates. KC should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2n$, provided $m \leq 20$.

Constraints:

$$KC \geq 2 \times NXY,$$

KC must be a multiple of L . The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC , counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF and C06EBF which perform the FFT.

9: XG(KC) – *real* array *Input/Output*

On entry: the NXY data points of the x series.

On exit: the real parts of the NG cross spectral estimates in elements XG(1) to XG(NG), and XG($NG + 1$) to XG(KC) contain 0.0. The y series leads the x series.

10: YG(KC) – *real* array *Input/Output*

On entry: the NXY data points of the y series.

On exit: the imaginary parts of the NG cross spectral estimates in elements YG(1) to YG(NG), and YG($NG + 1$) to YG(KC) contain 0.0. The y series leads the x series.

11: NG – INTEGER *Output*

On exit: the number of spectral estimates, $[L/2] + 1$, whose separate parts are held in XG and YG.

12: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $NXY < 1$,
 or $MTXY < 0$,
 or $MTXY > 2$,
 or $PXY < 0.0$,
 or $PXY > 1.0$,

or $MW < 1$,
 or $MW > NXY$,
 or $PW < 0.0$ and $MW \neq NXY$,
 or $PW > 1.0$ and $MW \neq NXY$,
 or $L < 1$,
 or $|IS| \geq L$.

IFAIL = 2

On entry, $KC < 2 \times NXY$,
 or KC is not a multiple of L ,
 or KC has a prime factor exceeding 19,
 or KC has more than 20 prime factors, counting repetitions.

IFAIL = 3

This indicates that a serious error has occurred. Check all array subscripts in calls to G13CDF. Seek expert help.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

G13CDF carries out an FFT of length KC to calculate the sample cross spectrum. The time taken by the routine for this is approximately proportional to $KC \times \log(KC)$ (but see routine document Section for further details).

9 Example

The example program reads two time series of length 296. It selects mean correction and a 10% tapering proportion. It selects a $2\pi/16$ frequency width of smoothing window, a window shape parameter of 0.5 and an alignment shift of 3. It then calls G13CDF to calculate the smoothed sample cross spectrum and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      G13CDF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER      NXYMAX, L, KC
      PARAMETER    (NXYMAX=300,L=80,KC=8*L)
      INTEGER      NIN, NOUT
      PARAMETER    (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real        PW, PXY
      INTEGER      I, IFAIL, IS, J, MTTY, MW, NG, NXY
*      .. Local Arrays ..
      real        XG(KC), YG(KC)
*      .. External Subroutines ..
      EXTERNAL    G13CDF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G13CDF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NXY
      IF (NXY.GT.0 .AND. NXY.LE.NXYMAX) THEN
        READ (NIN,*) (XG(I),I=1,NXY)
```

```

      READ (NIN,*) (YG(I),I=1,NXY)
*      Set parameters for call to G13CDF
*      Mean correction and 10 percent taper
      MTXY = 1
      PXY = 0.1e0
*      Window shape parameter and zero covariance at lag 16
      PW = 0.5e0
      MW = 16
*      Alignment shift of 3
      IS = 3
      IFAIL = 0
*
      CALL G13CDF(NXY,MTXY,PXY,MW,IS,PW,L,KC,XG,YG,NG,IFAIL)
*
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          Returned sample spectrum'
      WRITE (NOUT,*)
      WRITE (NOUT,*)
+      Real      Imaginary      Real      Imaginary      Real      Imaginary
+
      WRITE (NOUT,*)
+ '      part      part      part      part      part      part'
      WRITE (NOUT,99999) (J,XG(J),YG(J),J=1,NG)
      END IF
      STOP
*
99999 FORMAT (1X,I3,F8.4,F9.4,I5,F8.4,F9.4,I5,F8.4,F9.4)
      END

```

9.2 Program Data

G13CDF Example Program Data

```

296
-0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
 0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
-0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
 0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
 1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
 0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
 1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
 0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
 1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
-1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
-0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
-0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
 1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
-0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
-2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
-0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
 0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
 0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
 0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
 0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
 0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
 0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
 0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
 0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
 0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
 1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
 0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
 0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262

```

```

53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0

```

9.3 Program Results

G13CDF Example Program Results

Returned sample spectrum

	Real part	Imaginary part		Real part	Imaginary part		Real part	Imaginary part
1	-6.1563	0.0000	2	-5.5905	-2.0119	3	-3.2711	-2.7963
4	-1.1803	-2.3264	5	-0.2061	-1.8132	6	0.3434	-1.1357
7	0.6200	-0.7351	8	0.5967	-0.3449	9	0.4523	-0.0984
10	0.2391	0.0177	11	0.1129	0.0402	12	0.0564	0.0523
13	0.0134	0.0443	14	-0.0032	0.0299	15	-0.0057	0.0148
16	-0.0057	0.0069	17	-0.0033	0.0038	18	-0.0011	0.0012
19	-0.0004	0.0001	20	-0.0004	0.0002	21	-0.0003	0.0001
22	-0.0001	0.0002	23	-0.0002	0.0003	24	-0.0002	0.0002
25	-0.0002	0.0000	26	-0.0004	0.0000	27	-0.0002	-0.0002
28	-0.0001	-0.0000	29	-0.0001	0.0002	30	-0.0001	0.0002
31	-0.0002	0.0003	32	-0.0002	0.0001	33	-0.0001	0.0000
34	-0.0000	-0.0000	35	0.0000	-0.0001	36	0.0001	-0.0001
37	0.0001	-0.0001	38	0.0001	-0.0001	39	0.0000	-0.0001
40	0.0000	-0.0001	41	0.0001	0.0000			
